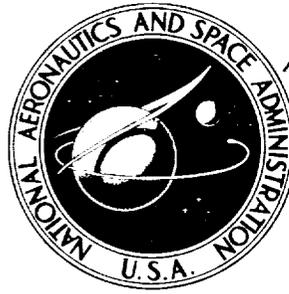


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Langley Research Center

Langley Station, Hampton, Va.

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SUMMARY

Some of the more commonly used real-gas¹ hypersonic-nozzle flow parameters have been calculated and charted for nitrogen in thermodynamic equilibrium; the study covered a range of stagnation pressures up to 1,000 atmospheres and stagnation temperatures from 1,800° R to 5,000° R. The flow parameters are presented in correction-factor form which indicates the magnitude of the departure of the various parameters from ideal-gas² (perfect gas) behavior. It is shown that significant error can be incurred in the values of the various flow parameters by using the ideal-gas relationship, particularly at the higher stagnation pressures. For the range of stagnation conditions considered in the study, the results presented will have direct application to such practical problems as calibration and performance estimation of hypersonic test facilities which use nitrogen as the test fluid.

INTRODUCTION

The Langley Research Center has presently under construction the Langley hypersonic nitrogen facility which is designed to operate at a Mach number of 17, at stagnation pressures up to 1,000 atmospheres, and at a stagnation temperature of approximately 4,000° F; this facility will have the capability of run times on the order of minutes. At these elevated stagnation conditions, the test gas ceases to behave as an ideal fluid, and the real-gas effects on the thermodynamic properties of the flow, both upstream and downstream of the normal shock, must be taken into account if the flow in the hypersonic nozzle is to be properly

¹The term "real gas" as used herein relates to the effects associated with high densities and also the variation of heat capacity with temperature.

²The term "ideal gas" as used herein refers to a perfect gas with constant ratio of heat capacities as defined in reference 1.

interpreted and analyzed; for example, these effects must be taken into account in the Mach number calibration and total temperature survey of the test region of the facility.

It is the purpose of this report to present the results of calculations, in easy-to-use graph form, which were made to determine the magnitude of the real-gas effects on hypersonic-nozzle flow parameters; the study covered a range of stagnation pressures up to 1,000 atmospheres and stagnation temperatures from 1,800° R to 5,000° R. In the present analysis, the nitrogen was at all times assumed to be in thermodynamic equilibrium; however, at moderately high temperatures the gas may be in vibrational nonequilibrium which will have an effect on the free-stream flow properties as indicated in reference 2.

SYMBOLS

a	velocity of sound
c_p	heat capacity at constant pressure
F	correction factor (ratio of real-gas flow parameter to ideal-gas flow parameter for $\gamma = 1.4$ at a given value of free-stream Mach number)
h	specific enthalpy
M	Mach number
p	pressure
R	gas constant
S	entropy
S/R	entropy, dimensionless
T	temperature
V	velocity
Z	compressibility factor
γ	ratio of specific heats
ρ	density

Subscripts:

i	ideal
p,T, ρ	pressure, temperatures, and density, respectively
ref	reference condition used in equation (1)
t,1	stagnation conditions upstream of normal shock
t,2	stagnation conditions downstream of normal shock
1	static conditions upstream of normal shock
2	static conditions downstream of normal shock

METHOD OF CALCULATION

The following assumptions were made concerning the expansion of the gas from the stagnation chamber downstream of the nozzle to the test section: the flow was assumed to expand at a constant value of entropy (determined by the stagnation pressure and temperature), and, at the test section, the free-stream gas was assumed to behave as a perfect or ideal gas with constant ratio of heat capacities. The latter assumption was felt to be a good approximation in view of the fact that all the results presented are for free-stream static pressures of 0.01 atmosphere or less and for static temperatures of less than 540° R.

In the region where the real-gas effects are significant, the state properties of the gas were based on the tabulated thermodynamic properties of nitrogen which are presented in references 3 and 4. An IBM 7090 electronic data processing system was used to obtain the numerical results presented. For the range of stagnation conditions considered in the study, the initial input thermodynamic data to the computer were obtained from reference 4. The tabulated thermodynamic data of reference 3 were programmed into the computer for a pressure range from 10 atmospheres to 0.01 atmosphere. Below 0.01 atmosphere, the thermodynamic properties of interest were computed by treating the nitrogen as a perfect gas.

Free-Stream Conditions

For a given stagnation temperature $T_{t,1}$ and stagnation pressure $p_{t,1}$, the flow in the nozzle is expanded isentropically to a free-stream static pressure p_1 of 0.01 atmosphere. The value of temperature T_1 at this point is calculated from the following ideal-gas equation:

$$\left(\frac{S}{R}\right)_{t,1} - \left(\frac{S}{R}\right)_{\text{ref}} = \frac{c_p}{R} \log_e \frac{T_1}{T_{\text{ref}}} - \log_e \frac{p_1}{p_{\text{ref}}} \quad (1)$$

At this point the subscript "ref" in equation (1) indicates the state properties of a pressure of 0.01 atmosphere and a temperature of 630° R.

Once T_1 is known, the other free-stream static-flow properties may be calculated from the following equations:

$$h_1 = c_p(T_1) \quad (2)$$

$$V_1 = \left[2(h_{t,1} - h_1) \right]^{1/2} \quad (3)$$

$$\rho_1 = \frac{p_1}{T_1 R} \quad (4)$$

and

$$a_1 = (\gamma R T_1)^{1/2} \quad (5)$$

From the point where the pressure p_1 was equal to 0.01 atmosphere, the flow in the system was further expanded by reducing the value of T_1 in 18° increments, still maintaining a constant value of $(S/R)_{t,1}$. The pressure at each temperature increment was found by using equation (1) with a reference condition of pressure equal to 0.01 atmosphere and entropy equal to $(S/R)_{t,1}$. The remaining flow properties at each particular point were calculated by using equations (2) to (5).

Conditions Just Downstream of Normal Shock

The thermodynamic properties of the gas downstream of the normal shock may now be determined by using the previously calculated free-stream flow properties in conjunction with the conservation equations and the equation of state. These equations are:

Conservation of mass -

$$\rho_1 V_1 = \rho_2 V_2 \quad (6)$$

Conservation of momentum -

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2 \quad (7)$$

Conservation of energy -

$$h_{t,1} = h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad (8)$$

and the equation of state -

$$p_2 = Z_2 \rho_2 R_2 T_2 \quad (9)$$

In order that the free-stream static condition downstream of the normal shock be found, equations (6) to (9) must be solved by an iterative procedure. First, an initial value of ρ_2 is assumed; the velocity V_2 is then calculated from equation (6). With the values of p_2 and h_2 which are found from equations (7) and (8), respectively, the temperature T_2 is interpolated from the programmed tabulated thermodynamic data. The value of ρ_2 is then calculated from equation (9) and compared with the assumed value of ρ_2 . The solution is found when the assumed value of ρ_2 is in agreement with the calculated value of ρ_2 .

Stagnation Condition Downstream of the Normal Shock

The determination of the stagnation conditions downstream of the normal shock involves an iteration for the assumed initial value of the stagnation temperature $T_{t,2}$. From the free-stream values of p_2 and T_2 , the value of $(S/R)_2$ was determined from programmed tabulated thermodynamic data. Downstream of the normal shock, the value of the static entropy $(S/R)_2$ is equal to the stagnation entropy $(S/R)_{t,2}$; therefore, the stagnation pressure $p_{t,2}$ is found from the programmed thermodynamic data with the known value of $(S/R)_{t,2}$ and the assumed value of $T_{t,2}$. The values of $T_{t,2}$ and $p_{t,2}$ are then used to determine a value of $h_{t,2}$ from the tabulated thermodynamic data. It can be seen from equation (8) that the stagnation enthalpy is constant throughout the flow system; thus, the solution is found when the value of $h_{t,1}$ is in agreement with $h_{t,2}$.

RESULTS AND DISCUSSION

The results of the calculations are presented in easy-to-use form in figures 1 to 9, wherein the correction factor for the various hypersonic-nozzle flow parameters is plotted against stagnation pressure for a range of stagnation temperatures. A correction factor is defined as the ratio of the real-gas flow parameter to the ideal-gas flow parameter at a given value of free-stream Mach

number; that is, $F_{T,1} = \frac{T/T_{t,1}}{(T/T_{t,1})_i}$. All the results, when presented in

correction-factor form, were found to be essentially independent of free-stream Mach number; however, because of small inaccuracies in the programmed data (caused by cross-plotting the thermodynamic data of ref. 3), there was some slight numerical variation in the calculated values of the various correction factors with Mach number. The variation in values was generally within ± 0.3 percent. In order to compensate partially for the variation, a numerical average of the

values for the particular correction factor was taken for a minimum of three different free-stream Mach numbers.

The use of the charts presented in figures 1 to 9 may be best shown by example. In the calibration of a hypersonic nozzle ($M > 10$), the pressure and temperature in the stagnation chamber and the pitot pressure in the test section are usually known from measurements; the real-gas Mach number in the test section is desired. For example, assume that $p_{t,1} = 500$ atmospheres, $T_{t,1} = 4,200^\circ \text{R}$, and the measured stagnation pressure downstream of the normal shock in the test section $p_{t,2}$ is 0.125 atmosphere. Then the real-gas ratio $p_{t,2}/p_{t,1}$ equals 2.5×10^{-4} atmosphere. From figure 7, it is seen that the value of $\frac{p_{t,2}/p_{t,1}}{(p_{t,2}/p_{t,1})_i}$ at the assumed stagnation conditions is 0.804; therefore, the value of $(p_{t,2}/p_{t,1})_i$ is $\frac{2.50 \times 10^{-4}}{0.804}$ or 3.11×10^{-4} . The real-gas Mach number in the test section based on this ideal-gas ratio may be obtained from reference 1; for this particular case, $M_1 = 16.11$. Other desired flow properties may be obtained in a similar manner.

In order that a check on the results presented be provided, values of $F_{\rho,1}$, $F_{p,1}$, and $F_{T,1}$ have been calculated for a few stagnation conditions considering only the effect of variable heat capacity due to temperature; a flow condition such as this would exist at low stagnation pressures, that is, $p_{t,1}$ approaches 0. A comparison of the results showed that values of the correction factors at this limiting condition agree within 0.2 percent.

Presented in figure 10 is a plot showing the approximate maximum Mach numbers obtainable just prior to flow condensation for a few stagnation temperatures. The values presented in this figure were obtained by using the real-gas calculations and the vapor pressure data presented in reference 3. It should be noted that it is possible to operate a hypersonic facility with some amount of supersaturation present in the flow without affecting the pitot pressure or free-stream static pressure (e.g., see ref. 5). A small decrease in the free-stream static temperature (slightly below the saturated vapor line) produces a decrease in the total temperature (increase in free-stream Mach number) required to operate a hypersonic facility. For example, at a stagnation pressure of 800 atmospheres and a stagnation temperature of $3,400^\circ \text{R}$, a 10° decrease in static temperature below the saturated vapor line will increase the free-stream Mach number from approximately 15.2 to 16.2.

As would be expected, a comparison of the real-gas correction factors for nitrogen with those for air, as presented in reference 6, showed that there was close agreement between the two gases, particularly in the lower range of stagnation temperatures ($T_{t,1} < 3,000^\circ \text{R}$); however, the deviation in values became more pronounced as stagnation temperature was increased because of dissociation of the oxygen in the air. For example, the difference between the values of the correction factor for the ratio of free-stream static pressure to stagnation pressure at a stagnation pressure of 200 atmospheres and a stagnation temperature of $3,000^\circ \text{R}$

is approximately 1 percent; however, the difference between the values at a stagnation temperature of $5,000^{\circ}$ R is approximately 4 percent.

CONCLUDING REMARKS

Calculations have been made to assess the magnitude of the real-gas effects on some of the more commonly used hypersonic-nozzle flow parameters for nitrogen in thermodynamic equilibrium. The results of the calculations are presented in easy-to-use correction-factor form and covered a range of stagnation pressures up to 1,000 atmospheres and stagnation temperatures from $1,800^{\circ}$ R to $5,000^{\circ}$ R. The results showed that by taking the real-gas effects of the thermodynamic properties into account in calculating the various flow parameters, a significant difference in the values as predicted by the ideal-gas relationship occurred.

As would be expected, a comparison of the real-gas correction factors for nitrogen with those for air showed that there was good agreement between the two gases, particularly in the lower range of stagnation temperatures (stagnation temperatures less than $3,000^{\circ}$ R); however, the deviation in the values became more pronounced as the stagnation temperature was increased because of dissociation of the oxygen in the air.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., August 5, 1963.

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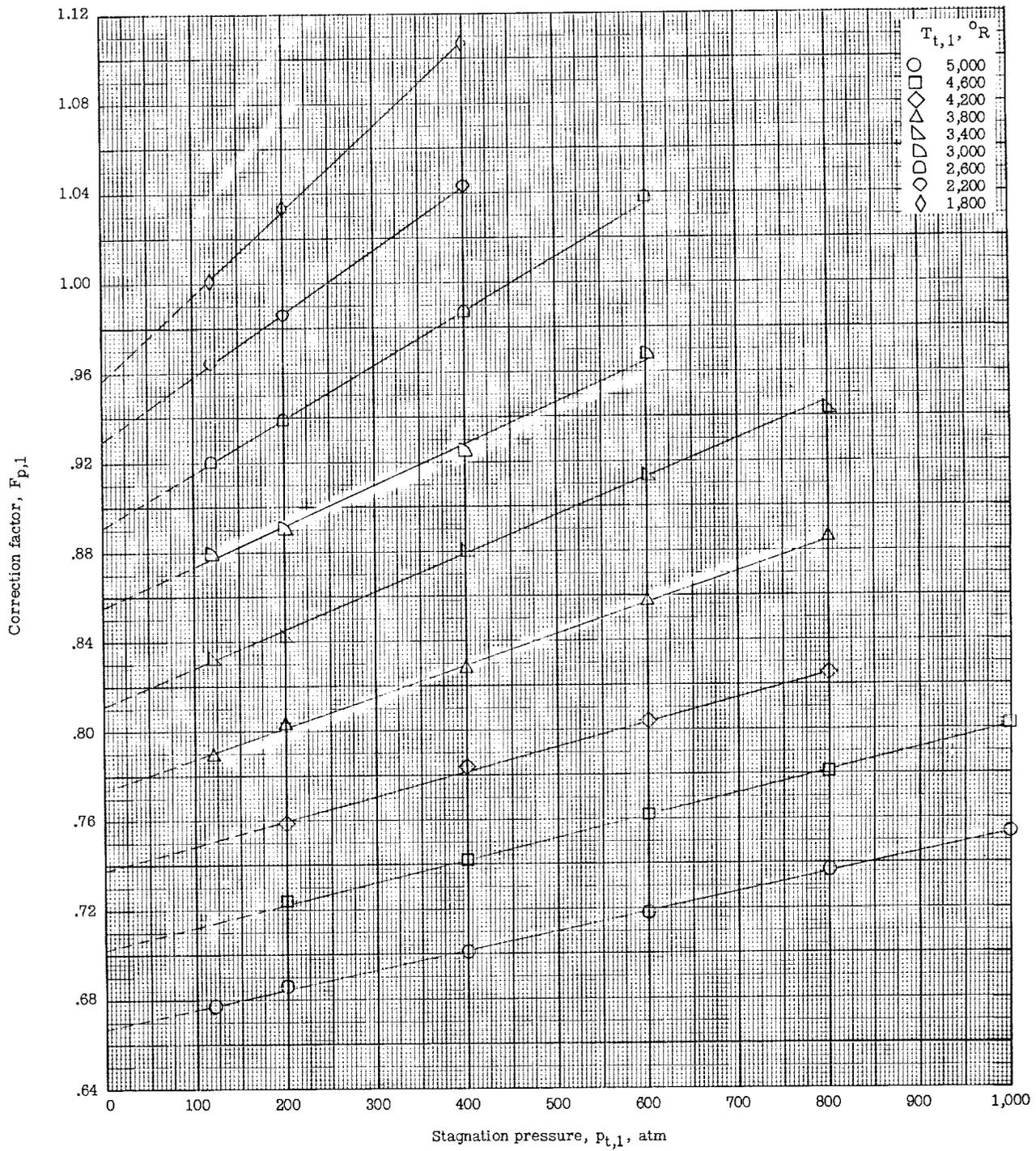


Figure 1.- Correction factor for $F_{p,1}$ as a function of stagnation pressure for various stagnation temperatures where $M_1 > 10$.

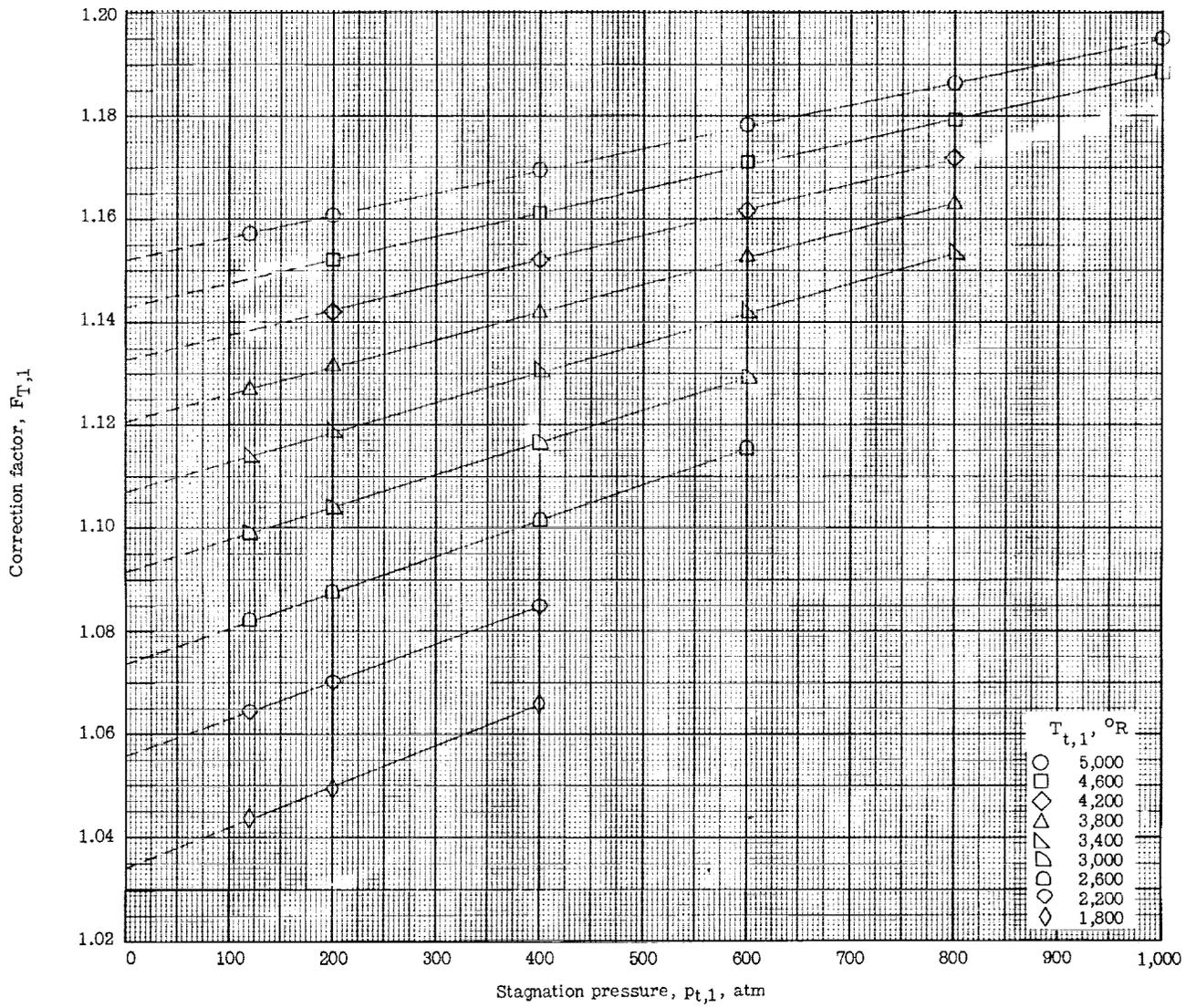


Figure 2.- Correction factor for $F_{T,1}$ as a function of stagnation pressure for various stagnation temperatures where $M_1 > 10$.

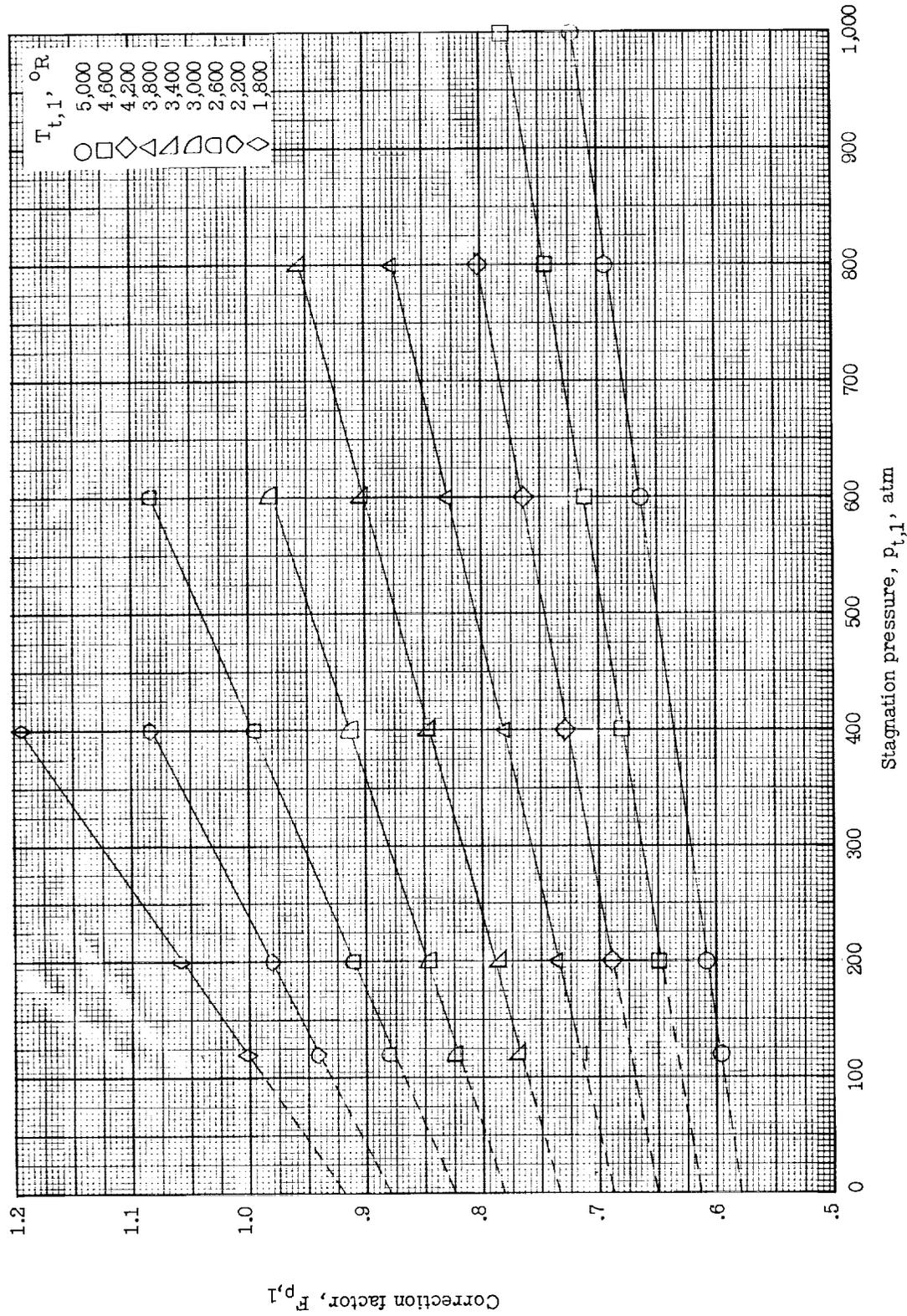


Figure 3.- Correction factor for $F_{p,1}$ as a function of stagnation pressure for various stagnation temperatures where $M_1 > 10$.

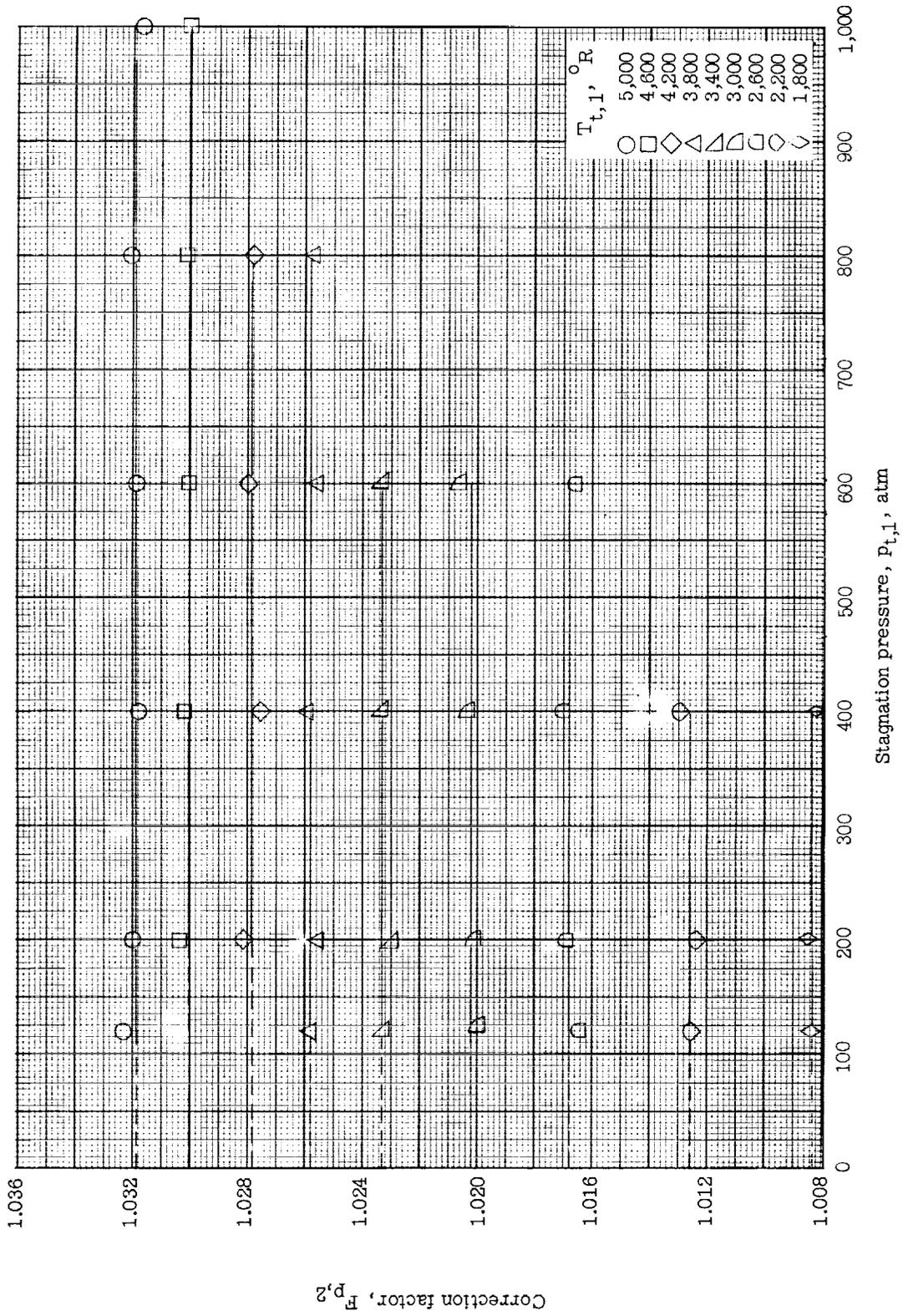


Figure 4.- Correction factor for $P_{p,2}$ as a function of stagnation pressure for various stagnation temperatures where $M_1 > 10$.

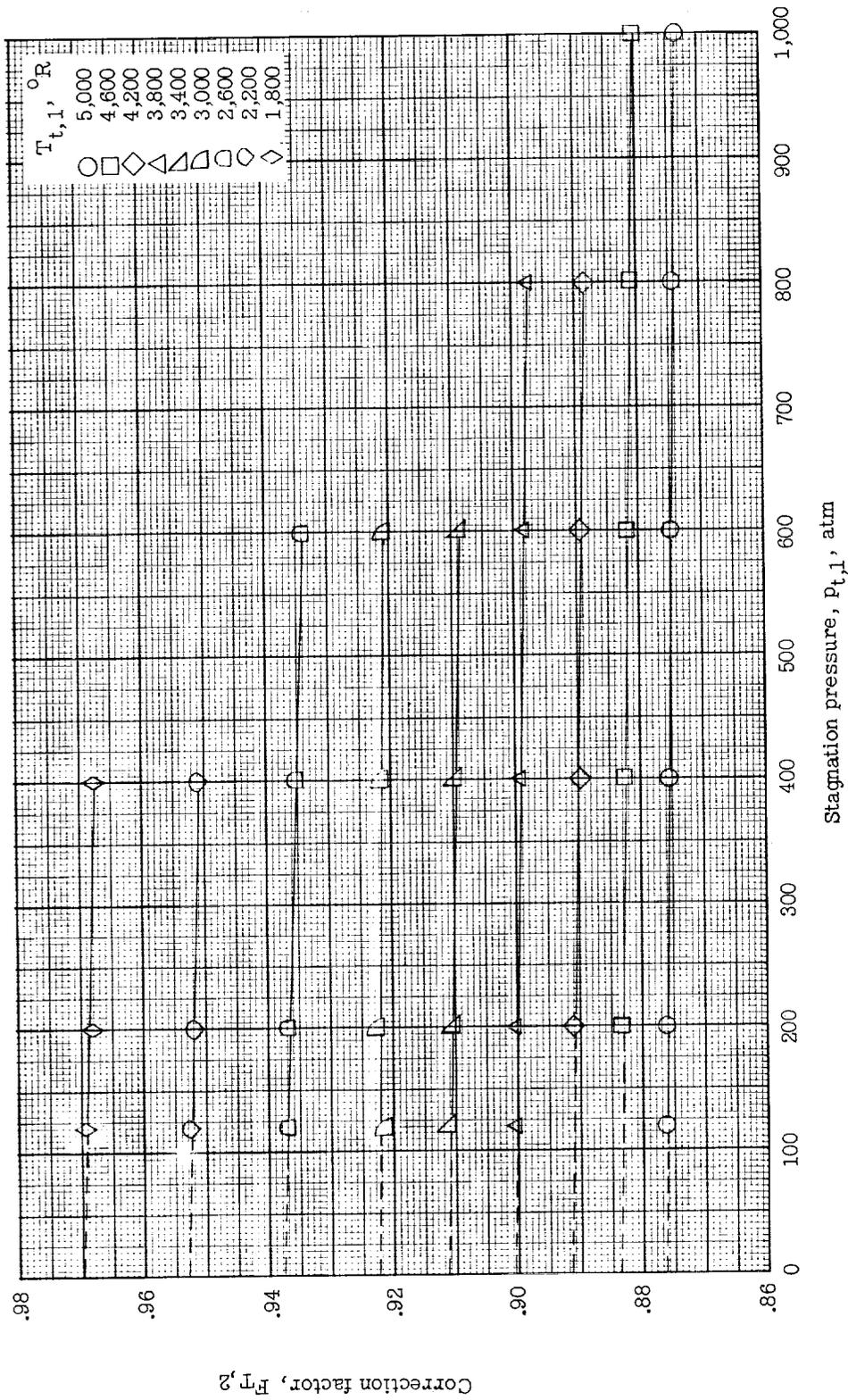


Figure 5.- Correction factor for $F_{T,2}$ as a function of stagnation pressure for various stagnation temperatures where $M_1 > 10$.

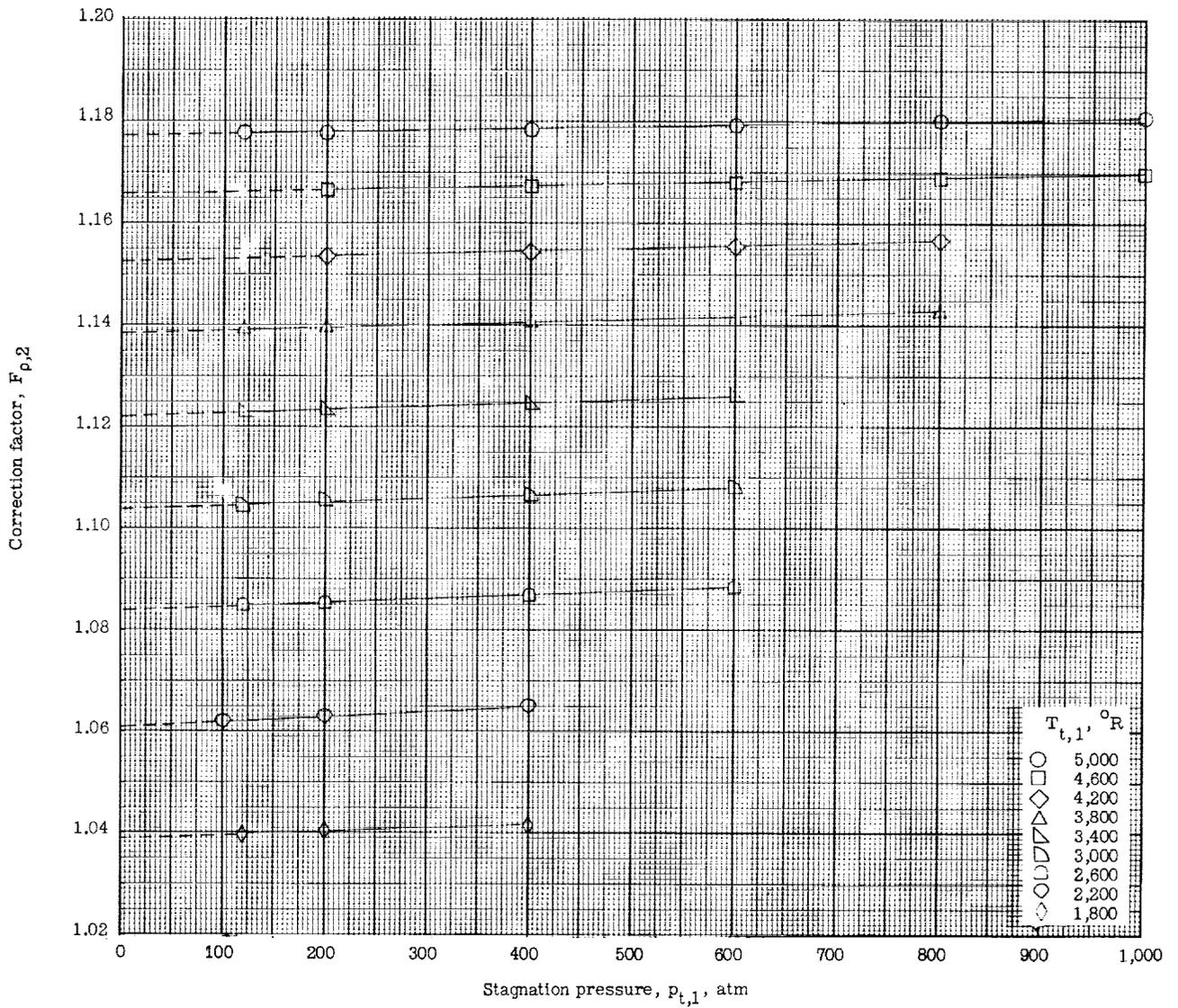


Figure 6.- Correction factor for $F_{p,2}$ as a function of stagnation pressure for various stagnation temperatures where $M_1 > 10$.

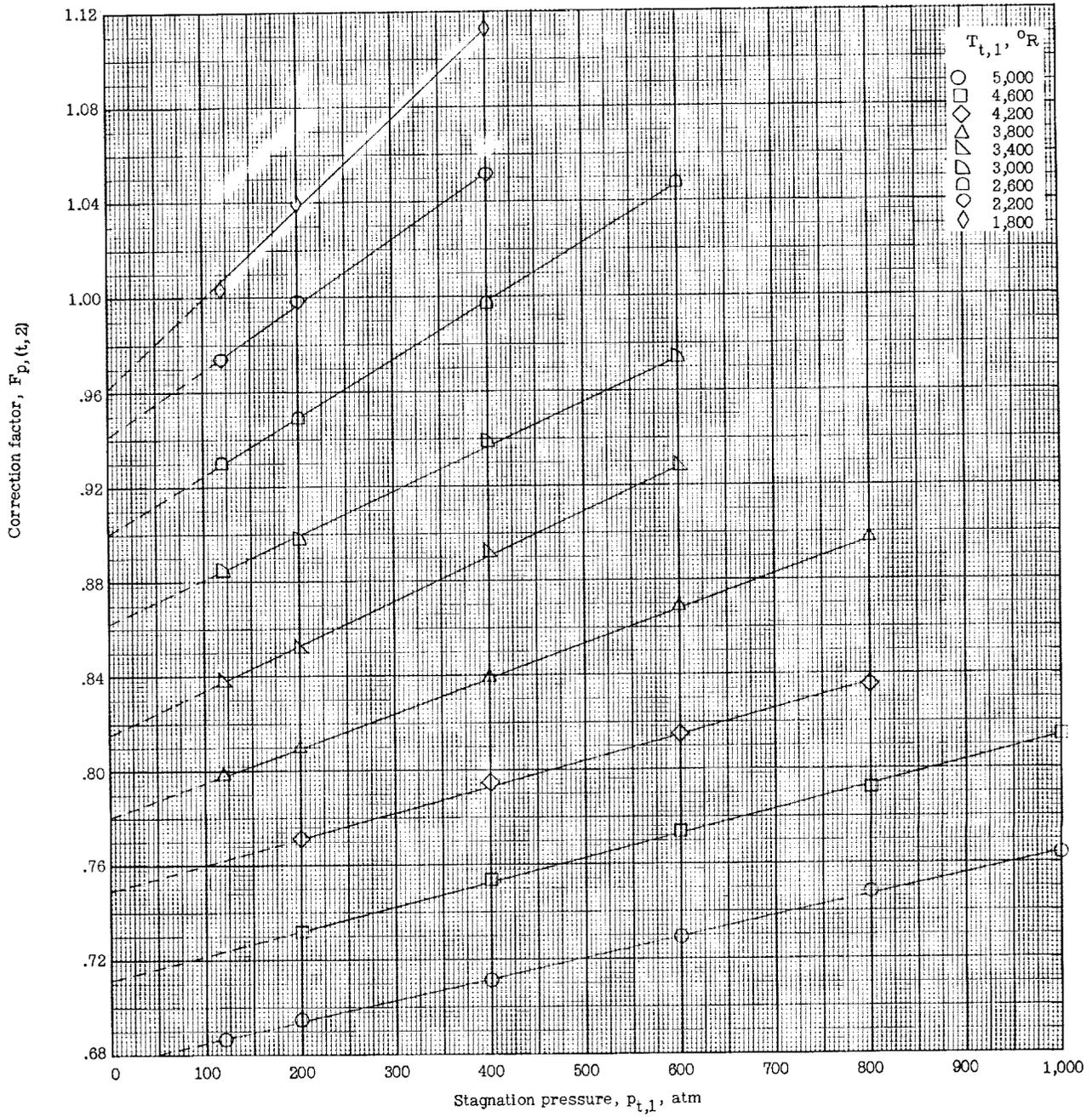


Figure 7.- Correction factor for $F_{p,(t,2)}$ as a function of stagnation pressure for various stagnation temperatures where $M_1 > 10$.

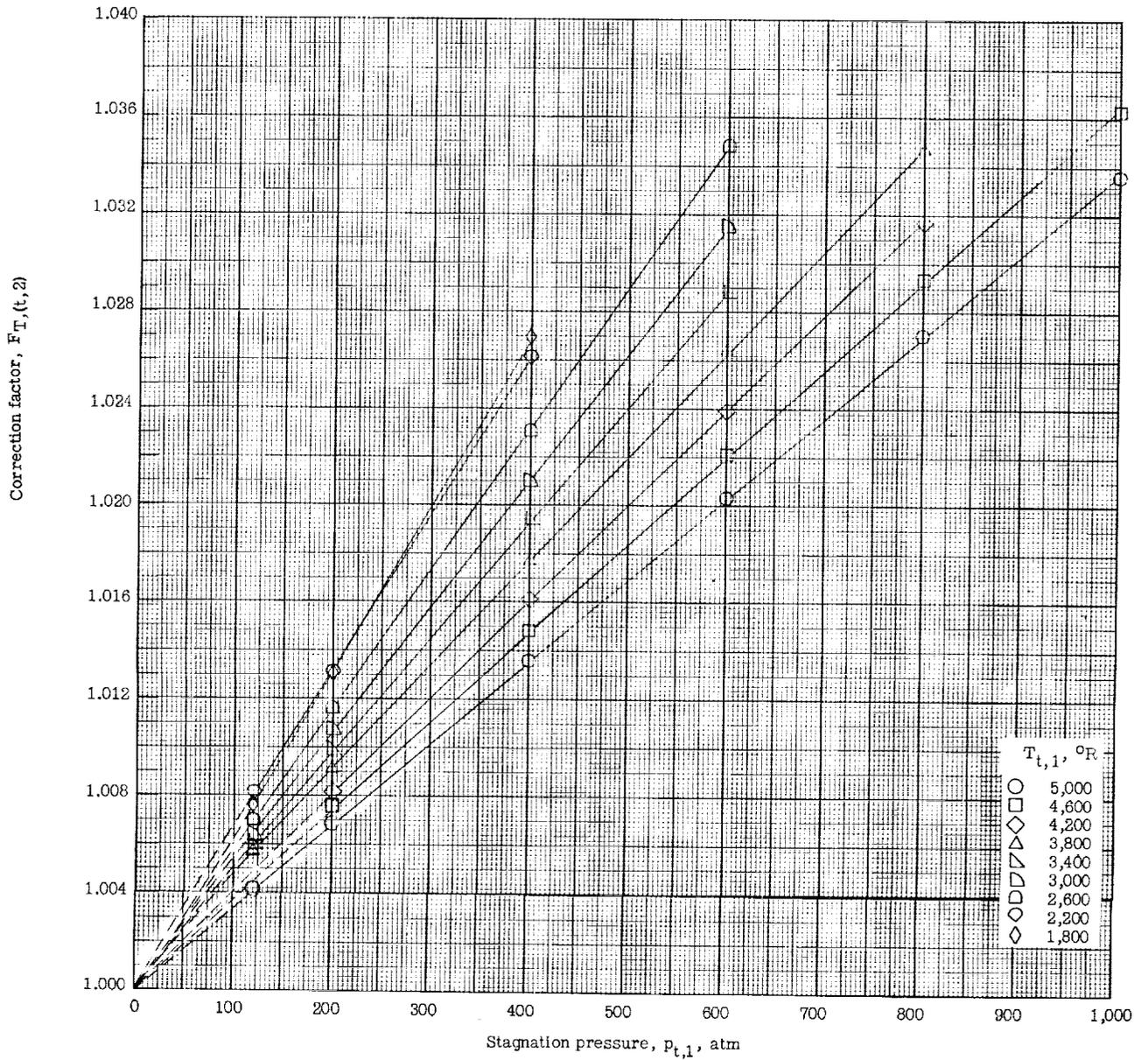


Figure 8.- Correction factor for $F_{T,t,2}$ as a function of stagnation pressure for various stagnation temperatures where $M_1 > 10$.

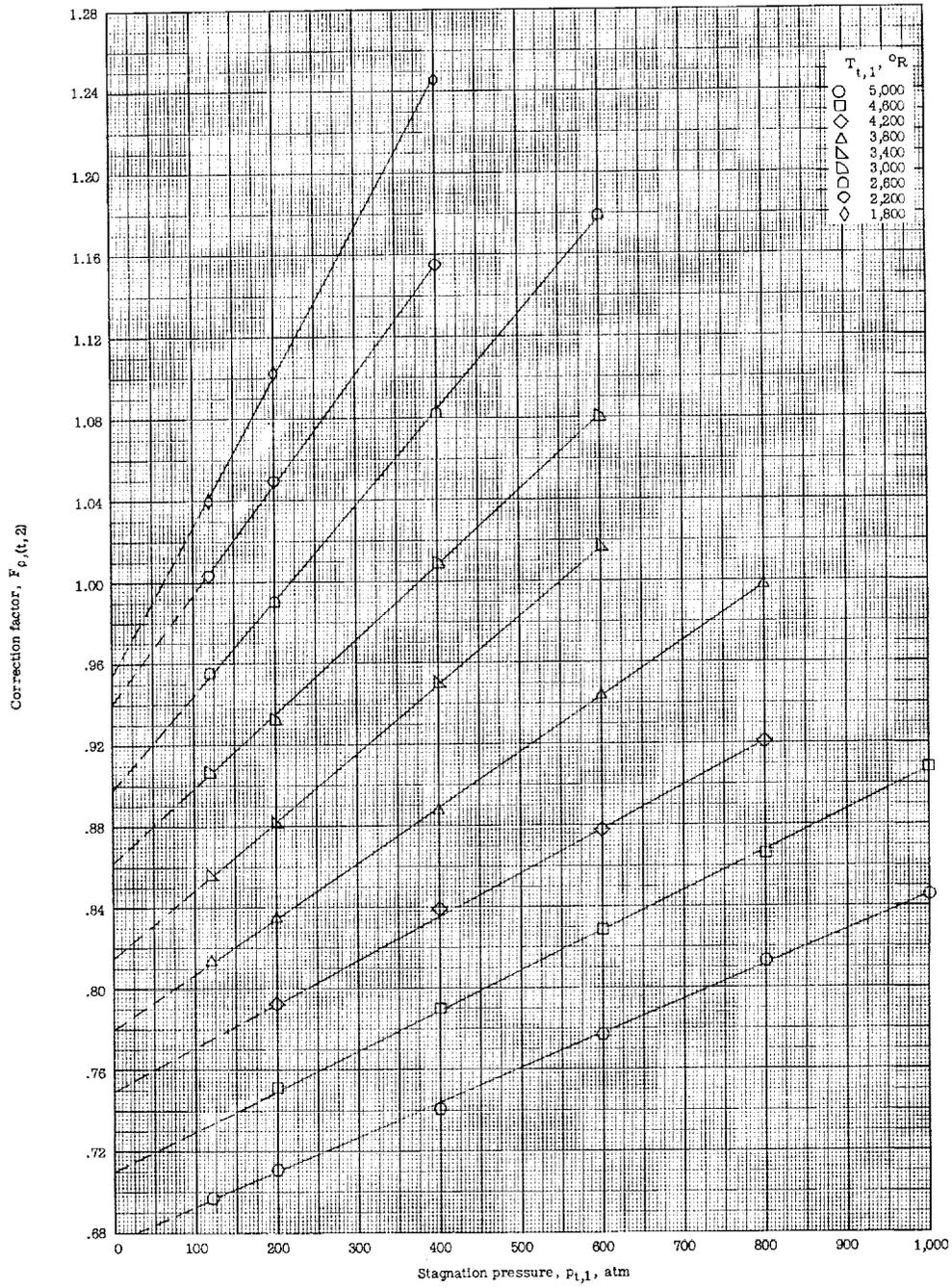


Figure 9.- Correction factor for $F_{\rho, (t, 2)}$ as a function of stagnation pressure for various stagnation temperatures where $M_1 > 10$.

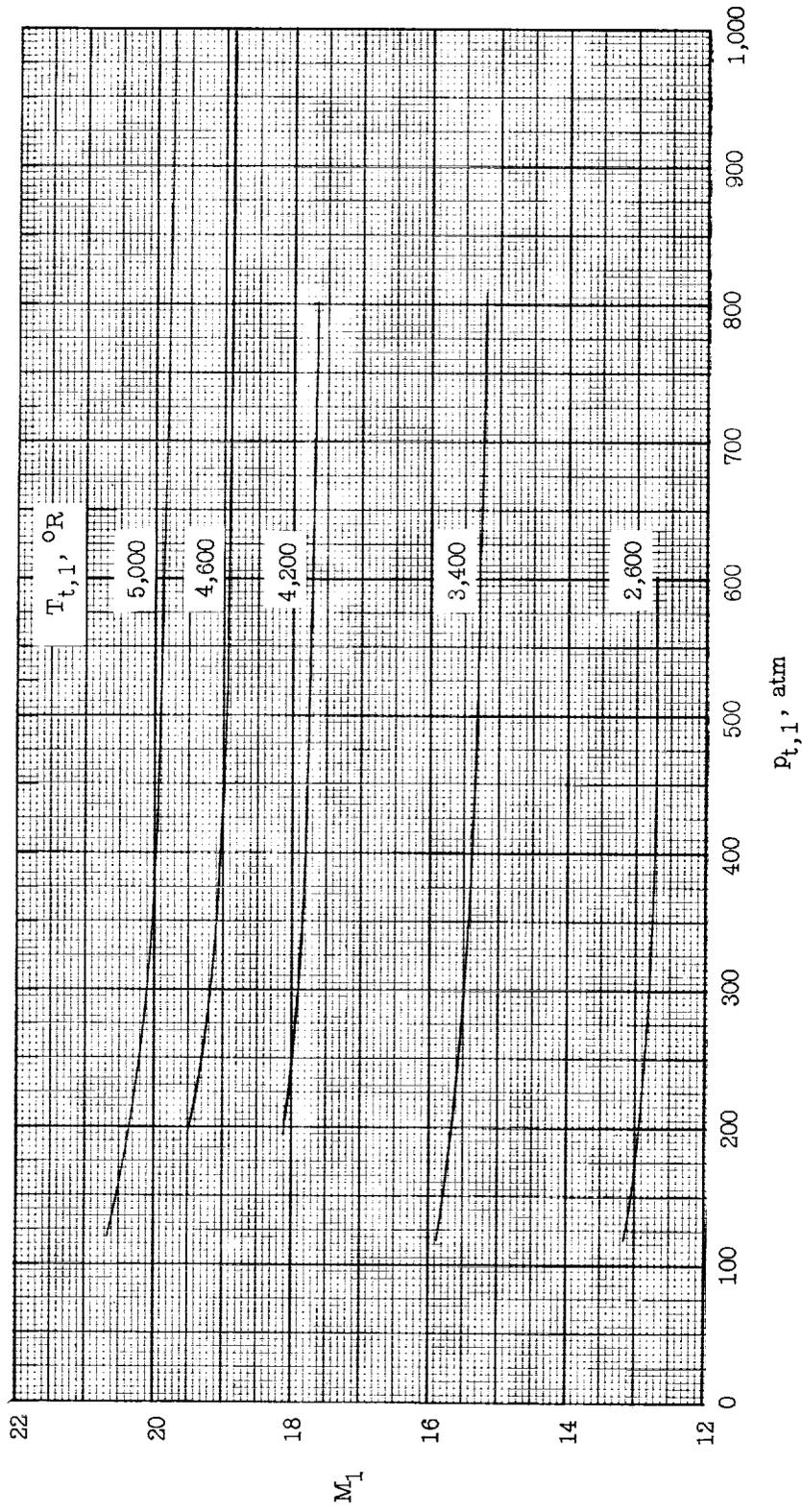


Figure 10.- Variation of Mach number at which condensation occurs for equilibrium conditions plotted against stagnation pressure for various stagnation temperatures.